

LIQUID FUEL DROPLET HEATING WITH INTERNAL CIRCULATION

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(Received 22 June 1977 and in revised form 4 November 1977)

Abstract—The internal circulation of a liquid droplet moving through a gas is analyzed in a parameter range relevant for a burning air-fuel spray mixture in a typical combustor. The analysis indicates axisymmetric quasi-steady liquid motion with a spherical core vortex surrounded by a viscous boundary layer and an internal wake. The vortex strength is determined as a function of the shear stress along the droplet surface. Furthermore, analysis indicates that the droplet heating is unsteady during its lifetime. The internal circulation is shown to one-dimensionalize effectively the heat conduction in the liquid core. The thermal boundary layer near the surface is also analyzed and it is shown that the droplet temperature field may be determined given the gas-liquid interface constraint.

NOMENCLATURE

a_0 - a_3 , constants used in boundary condition (14c);	V , $= \left(\frac{\partial T}{\partial Y_i} - C_1 \right)$,
A , strength of the Hill's vortex as used in (2);	variable defined in the thermal
b_0 - b_3 , constants related to a_0 - a_3 , given by (A2);	boundary layer;
C_1 , constant used in (21b);	W , transformed dimensionless vorticity
C_2 , constant used in (26b);	defined by (12a);
C_p , specific heat at constant pressure;	X , transformed θ co-ordinate defined by
D , diameter of the droplet;	(12c);
$f(\theta)$, related to $F(\theta)$ by (14b);	X_{∞} , $= 4/3$, value of X at the rear stagnation
$F(\theta)$, shear stress distribution at the interface;	point;
$g(Y)$, an unknown function of Y as used in (14a);	X_0 , value of X at the separation point;
$g_1(\phi), g_2(\phi)$, dimensionless function of ϕ defined by (25);	y , dimensionless distance defined by (10);
h_z , $= Ar^5 \cos^5 \theta / 2q_1$, scale factor in streamwise direction;	Y , transformed y co-ordinate defined by (12b).
h_{θ} , $= 1/(r \sin \theta q_1)$, scale factor normal to streamline in an azimuthal plane;	
h_r , $= r \sin \theta$, scale factor in azimuthal direction;	
K_1 , $= U_{\infty} / AR^2$, ratio of characteristic velocity in gas to that in liquid;	
p , $= r/R$, dimensionless distance;	
Pe , Peclet number;	
Pr , Prandtl number;	
\vec{q} , velocity vector;	
q_1 , velocity in streamwise direction;	
r , radial distance in spherical polar co-ordinate system;	
R , radius of droplet;	
Re , Reynolds number;	
t , time;	
T , temperature;	
T_0 , initial temperature;	
$T_{b,p}$, boiling point of liquid;	
u , dimensionless velocity defined by (10);	
U_{∞} , free stream gas velocity relative to droplet;	
U_b , $= AR^2$, characteristic liquid velocity;	

Greek symbols

α , $= \lambda / \rho C_p$, thermal diffusivity;
β , $= \pi - \theta$;
δ , boundary-layer thickness;
δ_w , thickness of the internal wake;
θ , tangential co-ordinate direction;
ν , $= \mu / \rho$, kinematic viscosity;
μ , viscosity;
ρ , density;
ψ , stream function;
ϕ , dimensionless stream function defined by (25);
ξ , $= (r^2 - R^2/2) / r^4 \cos^4 \theta = \text{constant}$, trajectories orthogonal to streamlines;
η , $= \text{constant}$, planes through the axis of symmetry;
τ , dimensionless time;
λ , thermal conductivity;
Ω , dimensionless vorticity defined by (11);
ϕ_0 , value of ϕ at the bounding streamline of the core;
ω , vorticity.

Subscripts

- o.* Hill's vortex solution;
- l.* perturbations;
- g.* gas phase;
- l.* liquid phase;
- t.* thermal boundary layer and internal thermal wake;
- 0.* initial value.

Superscripts

- \cdot dimensionless quantities.

1. INTRODUCTION

THERE have been a number of studies, both theoretical and experimental, of isolated droplet vaporization and combustion with spherical symmetry in a stagnant ambient gas [1]. These studies have provided useful insights to the problem. However, the practical situation usually involves vaporization of droplets in a convective gas stream. Some empirical correlations exist [2-4] which account for the convective motion in the gas phase and express the vaporization rate as a modification to the spherically symmetric case. These correlations are not very satisfactory especially for transient situations [5].

In many practical high pressure combustors, the Reynolds number based upon relative gas-droplet velocity is large [$O(100)$] compared to unity for a significant part of the droplet lifetime [6]. This is particularly so for larger droplets and for those in the outer part of the liquid fuel spray where relative gas-droplet velocity is larger. This high Reynolds number in the gas phase has been demonstrated to imply that the shear stress at the gas-liquid interface is large enough to induce internal circulation (in the absence of any surface-active impurities), with Reynolds number large [$O(100)$] compared to unity, in the liquid phase [6]. This liquid motion would be important in heat and mass transfer (for multi-component fuel droplets) within the droplet and thereby modify the vaporization rate.

The theoretical prediction of vaporization rate, taking into account the liquid phase convective motion, involves the solution of coupled equations of motion, energy and concentration in the gas and liquid phases. The coupling between the conservation equations in the two phases occurs at the gas-liquid interface. The present paper deals with only the liquid phase part of the overall problem which has been uncoupled by providing the necessary conditions at the gas-liquid interface. It is shown that the liquid phase motion can be treated as quasi-steady and the analysis for the motion has been conducted. Droplet heating is shown to be unsteady and the relevant energy equation is analyzed. The importance of liquid motion for heat and mass transfer (for multi-component fuels) within the droplet is indicated. The analysis outlined here can be extended to the coupled problem of vaporization with some modifications. The analysis is intended to provide useful insights to the liquid phase processes.

The high Reynolds number flow induced within a spherical droplet and gas bubble have been studied by Chao [7, 8], Moore [9], and Harper and Moore [10]. Most of the studies on a liquid droplet, and in particular, the theoretical study of Harper and Moore [10] are concerned with the steady motion of a liquid droplet in another liquid of comparable density and viscosity. In their problem, the first approximation to the flow was an inviscid solution which for the droplet interior was shown to be a Hill's spherical vortex whose strength was determined by requiring the continuity of tangential velocity at the droplet surface. The viscous boundary layer due to unmatched shear stress at the interface was shown to perturb the velocity field only slightly at high Reynolds numbers and permitted the linearization of the boundary layer equations. These equations were solved and a higher order correction to the strength of the Hill's vortex was determined. In the present problem of a liquid droplet motion in a convective gaseous environment, the density and viscosity in the two phases are different by orders of magnitude. Therefore an inviscid solution cannot be used as a first approximation as was done in [10]. For steady motion, it is shown following Batchelor [11] that the droplet core motion is still a Hill's vortex for high Reynolds number flow. The approximate strength of the vortex is not the strength of the inviscid solution and has to be determined. However, it is still possible to linearize the boundary-layer equation by realizing that the velocity perturbation to the Hill's vortex field is small even though the vorticity perturbation is large.

Even though these important physical differences exist, the mathematical analysis for the boundary layer, turning regions near the stagnation points and the internal wake near the axis of symmetry is primarily based on the ideas by Harper and Moore [10].

For droplet heating, the thermal boundary layer is shown to be quasi-steady and the relevant energy equation is determined. A procedure similar to the one for viscous boundary layer is outlined for its solution. By comparison of the relevant characteristic times, the heating of the droplet core is shown to be modeled well by a one-dimensional, unsteady energy equation in streamline coordinates. The relation between the thermal boundary layer and the thermal core is clearly indicated. The mathematical equations and procedure for the thermal core were discovered to be similar to those used by Brignell [12] for his problem of solute extraction from an internally circulating liquid drop, although they were developed independently. On the arguments that the boundary layer is thin and that the diffusion in the core is the rate controlling process, Brignell could solve the problem of mass transfer from the droplet by neglecting the existence of the boundary layer and solving the diffusion equation for the core. He imposed the droplet surface concentration as the boundary condition for the outer boundary of the core, thus eliminating the coupling between the boundary layer and the core without causing any significant

errors in the mass-transfer problem. But in the present problem of droplet vaporization, the fuel vapor concentration at the droplet surface depends exponentially on the surface temperature which, therefore, sensitively controls the vaporization rate. So such a simplification as that of Brignell is not possible for the present problem and the coupling between the thermal boundary layer and the core is essential. This coupling is clearly shown in this paper although the coupled equations are not solved for the reasons stated in that section. The diffusion equation for the core is solved for the two cases where the normalized boundary conditions are given for either (1) the temperature gradient or (2) the temperature (this second case is the same as in [12]). It is demonstrated in both cases that the characteristic diffusion times are in fact reduced by an order of magnitude due to the convective vortex motion.

In Section II, the liquid droplet motion is considered. Section III deals with the energy equation and droplet heating. Section IV is devoted to the discussion of the above analysis in the context of the coupled problem of droplet vaporization in a convective gaseous environment.

It should be understood that this paper does not provide any final results which can be used directly on the vaporizing fuel droplet problem. It only provides an extension of the previous works in the context of the present problem and a basis and direction for the continued work on the overall vaporization problem.

2. DROPLET MOTION

There is an initial period during which the droplet motion is unsteady and develops to a full vortex strength. Also, for a vaporizing droplet, the change in droplet radius will cause the droplet motion to change. Characteristic time for a droplet to establish a steady motion can be roughly estimated as follows:

$$\tau_{\text{vorticity diffusion}} = R^2/\nu_l,$$

which for a typical fuel (ethanol) droplet of radius $25\mu\text{m}$ at 298K is of the order of 0.4ms . Once the vortex motion is started, the characteristic length scale for the diffusion of vorticity is reduced to about one third the droplet radius (for Hill's vortex [13]) because the vortex center is located nearer to the surface than the center of the sphere. This reduces the characteristic vorticity diffusion time by an order of magnitude to about $40\mu\text{s}$. A characteristic time of this order is indicated by Savic's [14] theoretical solution for low Reynolds number flow to encompass the total mass starting from rest.

The lifetime of a vaporizing droplet, a characteristic time relevant for comparison with other characteristic times, can be roughly estimated, by using empirical correlation [3] for a vaporizing droplet in a convective field, as follows:

$$\tau_{\text{life}} = R^2/k(1 + 0.3Re^{1/2}Pr^{1/3})$$

which is of the order of 2ms for a typical value of evaporation constant $k = 10^{-7}\text{m}^2/\text{s}$ for a vaporizing

droplet and Reynolds number based on the relative gas velocity, $Re = 100$ and for the droplet radius of $25\mu\text{m}$ as before.

From a comparison of the droplet lifetime and the vorticity diffusion time, it can be concluded that the droplet motion can be treated as quasi-steady because the latter is smaller than the first by an order of magnitude. In passing, it may be indicated that the gas phase can also be treated as quasi-steady because the characteristic time which is the residence time in the boundary layer (R/U_∞) is of the order of $1\mu\text{s}$ for a relative velocity of 25m/s , and is three orders of magnitude smaller than the lifetime.

(A) Core motion

For high Reynolds number flow, the droplet core motion away from the thin viscous boundary layer near the droplet surface, is essentially inviscid. Following Batchelor [11], it can be shown that the axisymmetric steady state core motion in a confined domain for an incompressible liquid is given by,

$$\omega/(r \sin \theta) = \text{constant}.$$

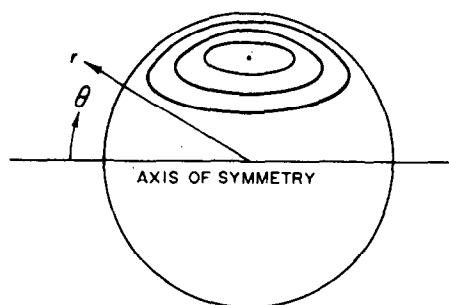


FIG. 1. Streamlines for the Hill's spherical vortex.

In the above, the spherical polar co-ordinate system is used and ω is the vorticity. In a spherical domain of the droplet, the core motion is a Hill's spherical vortex [13] for which the stream function is given by (see Fig. 1),

$$\psi_0 = -\frac{1}{2}Ar^2(R^2 - r^2)\sin^2 \theta, \quad (1)$$

and

$$\omega_0/(r \sin \theta) = 5A. \quad (2)$$

The constant A , used above, represents the strength of the Hill's vortex and has to be determined. It should be emphasized again that in the Harper and Moore's problem [10] the strength could be determined to a first approximation as an inviscid solution and $A = 3U_\infty/(2R^2)$ was known. This is the main difference between [10] and this section of the present paper.

The components of the velocity vector for the Hill's vortex are,

$$q_{\theta} = -A(R^2 - 2r^2)\sin \theta, \quad (3)$$

$$q_{r} = A(R^2 - r^2)\cos \theta. \quad (4)$$

It should be noted that the characteristic velocity in the liquid droplet is $U_l = O(AR^2)$ and the characteristic vorticity is of $O(AR)$.

(B) *Liquid boundary-layer analysis*

The techniques of the boundary-layer analysis are similar, in principle, to the Harper and Moore's [10] analysis. The boundary-layer velocity field is expressed as a perturbation to the Hill's vortex solution valid in the core. Therefore in the boundary layer,

$$\left. \begin{aligned} \bar{q} &= q_o + \bar{q}_1 \\ \bar{\omega} &= \omega_o + \bar{\omega}_1 \\ \psi &= \psi_o + \psi_1 \end{aligned} \right\} \quad (5)$$

Here the quantities with subscript "o" refer to the Hill's vortex solution and with subscript "1" refer to the perturbations. To match the boundary layer smoothly with the core, we require

$$\left. \begin{aligned} \bar{q}_1 &= 0 \\ \bar{\omega}_1 &= 0 \end{aligned} \right\} \quad (6)$$

at the inner edge of the boundary layer.

The steady-state continuity and momentum equation can be written for the boundary layer. The continuity equation is,

$$\nabla \cdot \bar{q}_1 = 0, \quad (7)$$

since the fluid is incompressible and \bar{q}_o satisfies $\nabla \cdot \bar{q}_o = 0$. Taking curl of the momentum equation and noting that $\bar{q} \cdot \nabla \bar{q} = \nabla(\bar{q}^2/2) - \bar{q} \times (\nabla \times \bar{q})$, we can write the momentum equation in the form,

$$v_1 \nabla^2 \bar{\omega}_1 + \nabla \times (\bar{q}_o \times \omega_1) + \nabla(\bar{q}_1 \times \omega_o) + \nabla \times (\bar{q}_1 \times \bar{\omega}_1) = 0. \quad (8)$$

Here use has been made of the fact that the Hill's vortex solution satisfies,

$$\nabla^2 \bar{\omega}_o = 0; \quad \text{and} \quad \nabla \times (\bar{q}_o \times \omega_o) = 0.$$

The momentum equation written above, in terms of $\bar{\omega}_1$, is non-linear. However, it can be linearized by making an order of magnitude estimate for the various quantities involved. In the momentum equation, the viscous terms will balance the inertia terms only if the non-dimensional boundary-layer thickness $\delta = O(Re^{-1/2})$. Here δ is non-dimensionalized with the length scale R . Therefore, for large Reynolds number in the liquid, $\delta \ll 1$ and, in the boundary layer, $\partial/\partial r = O(1/\delta)$. For a high Reynolds number flow in a confined domain, the boundary layer is a region where vorticity gradients are large compared to unity. It is assumed, therefore, that the perturbation vorticity ω_1 when non-dimensionalized by (AR) is of the order of unity, which is the same order as the Hill's vorticity in the core. This assumption will be examined *a posteriori* when the strength of the Hill's vortex has been determined. Therefore,

$$\omega_1 \approx \frac{\partial q'_r}{\partial r} = O(1),$$

$$q'_{\theta, r} = O(\delta),$$

and

$$q'_r = O(\delta^2).$$

Primes, in the above, denote the non-dimensional

quantities and the velocity has been non-dimensionalized with the characteristic velocity AR^2 .

It is the recognition of the fact that the velocity perturbation is small even though the vorticity perturbation is not small which allows us to extend the Harper and Moore analysis into a parameter domain well beyond that originally intended. An order of magnitude estimate for each of the terms in equation (8) can now be made. Retaining terms of $O(1)$, we can write down the linearized momentum equation in the axi-symmetric boundary layer,

$$v_1 \left[\frac{\partial^2 \omega_1}{\partial r^2} - A(R^2 - r^2) \cos \theta \frac{\partial \omega_1}{\partial r} + A(R^2 - 2r^2) \sin \theta \frac{\partial \omega_1}{r \partial \theta} + \frac{AR^2 \cos \theta}{r} \omega_1 \right] = 0. \quad (9)$$

Here, use has been made of the Hill's vortex solution.

Now, the quantities in the above equation (9) are non-dimensionalized. In particular, define,

$$\left. \begin{aligned} \delta^2 &= 2/Rc_l = v_1 AR^3, \\ q_{\theta, r} &= AR^2 \cdot \delta \cdot u \\ R - r &= R\delta v. \end{aligned} \right\} \quad (10)$$

Define the non-dimensional vorticity as

$$\Omega = \frac{\omega_1}{AR} = \frac{\partial u}{\partial v}. \quad (11)$$

A further transformation (similar to the one in [10]) of the dependent and independent variables is made below. Define,

$$W = \frac{\Omega}{\sin \theta} \quad (12a)$$

$$Y = v \sin^2 \theta; \quad 0 \leq Y < \mathcal{Y} \quad (12b)$$

$$X = \frac{1}{12} \cos 3\theta - \frac{3}{4} \cos \theta + \frac{2}{3}; \quad 0 < X < X_e = \frac{4}{3}. \quad (12c)$$

The momentum equation (9) now reduces to the well known form of the diffusion equation,

$$\frac{\partial^2 W}{\partial Y^2} = \frac{\partial W}{\partial X}. \quad (13)$$

Here $Y = 0$ corresponds to the droplet surface and $Y \rightarrow \infty$ to the edge of the boundary layer. Similarly $X = 0$ corresponds to the front stagnation point region and $X = X_e = \frac{4}{3}$ to the rear stagnation point region. It should be indicated that the boundary-layer equation (13) is not valid near the front and the rear stagnation point regions where some of the neglected terms in the momentum equation are of the same order as the retained terms and the boundary-layer approximation is no longer valid. Boundary condition to the above equation at $Y = 0$ is obtained by requiring continuity of shear stress at the gas-liquid interface. Therefore,

$$(1) \text{ at } Y = 0, \quad W = -3 + \frac{F(\theta) \sin \theta}{\mu_l \cdot AR},$$

where $F(\theta)$ is the shear stress distribution due to the relative motion of the gas with respect to the droplet. Here it is assumed that there are no surface-active impurities present and that any forces due to surface

tension gradient are negligible compared to the shear stress. At the edge of the boundary layer we have,

$$(2) \text{ as } Y \rightarrow \infty, \quad W \rightarrow 0.$$

This condition is needed to match the solution smoothly to the Hill's vortex solution.

Another boundary condition is needed at some value of $\theta = \text{constant}$ (or $X = \text{constant}$). Let us assume that,

$$(3) \text{ at } X = 0^+, \quad W = g(Y) \quad (14a)$$

where $g(Y)$ is some function of Y which is yet unknown and will depend upon the overall flow pattern, near the stagnation point regions and in the internal wake. Note that the boundary condition has been applied at some X , slightly away from the front stagnation point.

Some explanation is needed about the boundary condition (1). In an actual situation of flow over a sphere, the gas phase boundary layer will separate at some value of $\theta = \theta_s$. Let us say the corresponding value of X is X_0 . From the separation point aft, up until the rear stagnation point, there will be a very small shear stress in the opposite direction due to reverse flow in the separated region. In this paper it is assumed that the shear stress is zero in the separated region. Note that the separation point is moved aft when the surface of the sphere is moving, as in this case, compared to the case of a solid sphere. The shear stress distribution at the droplet surface up until the point of separation was taken for flow over a sphere [15, 16] and expressed in the form given below.

$$F(\theta) = C_1 \mu_g U_\infty f(\theta)/R; \quad 0 < \theta \leq \theta_s \\ = 0; \quad \theta_s \leq \theta < \pi, \quad (14b)$$

where C_1 is a constant and $f(\theta)$ is some other function of θ . Now the boundary condition (1) at $Y = 0$ can be expressed in terms of X as below.

$$\text{At } Y = 0, \quad W = -3 + \frac{U_\infty}{AR^2} (a_0 + a_1 X + a_2 X^2 + a_3 X^3); \\ 0 < X \leq X_0 \\ = -3; \quad X_0 \leq X < X_e. \quad (14c)$$

Here the constants $a_0, a_1 \dots$ etc. are obtained by a polynomial fit of the quantity $[C_1 \cdot (\mu_g/\mu_l) (f(\theta)/\sin \theta)]$ in terms of X .

For simplification we define,

$$K_1 \equiv \frac{U_\infty}{AR^2}, \quad (15)$$

which is the ratio of the characteristic velocity in the gas (the free stream velocity) to the characteristic velocity in the liquid.

The solution of the diffusion equation (13) subject to the above boundary conditions can now be written as

$$W = \frac{1}{2(\pi X)^{1/2}} \int_0^\infty g(Y') [\exp(-(Y - Y')^2/4X) - \exp(-(Y + Y')^2/4X)] dY' - 3\phi_a(X, Y) + K_1 \phi_b(X, Y), \quad (16)$$

where $\phi_a(X, Y) = \text{erfc}(Y/2X^{1/2})$ and $\phi_b(X, Y)$ is a long function involving integrals of complementary error functions and is given in Appendix A.

In the above solution $g(Y)$ and K_1 are still unknowns. This is similar to equation (3.20) in [10] where $\gamma(Y)$ and C are the unknowns. However, here K_1 represents the full vortex strength while C represents only the perturbation to the vortex strength. To determine these unknowns, an approach similar to the one in [10] was taken in which a condition in the form of an integral equation is obtained. To deduce this condition the flow pattern near the rear stagnation point, where the flow turns and enters the internal wake, and in the internal wake (see Fig. 2) should be analyzed.

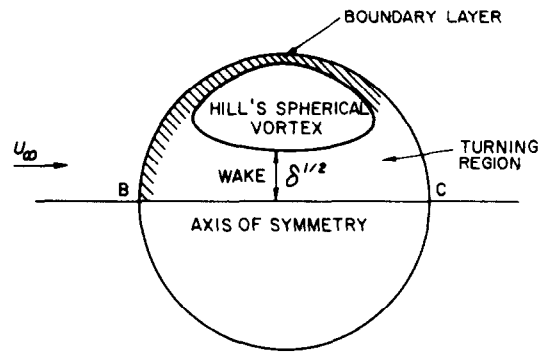


Fig. 2. Schematic diagram of the liquid droplet motion with viscous boundary layer, core and internal wake.

An order of magnitude analysis for the rear stagnation point region was made by Moore [9] and used by Harper and Moore [10]. A similar analysis for the magnitude of the terms in the exact equation of motion given by equation (8) estimated according to the boundary-layer approximation produced the same conclusion as in [10]. Results of this analysis are reproduced here for the sake of completeness. From this order of magnitude analysis it can be shown that the turning region near the rear stagnation point is of size $O(\delta^{1/3})$ or $O(Re^{-1/6})$. Also the perturbation vorticity in this region is $O(\beta)$ or $O(Re^{-1/6})$, where $\beta = \pi - \theta$, and therefore the perturbation velocity is $O(Re^{-1/3})$. The velocity of the Hill's vortex solution in this region is $O(\beta)$ or $O(Re^{-1/6})$. Therefore, the perturbation velocity is smaller than the Hill's vortex velocities by a factor of $O(Re^{-1/6})$. Thus the streamlines are only slightly perturbed for the Hill's vortex solution and the assumption $\psi \approx \psi_0$ in this region is correct to a first order approximation. Also the viscous forces are $O(\beta^5)$ or $O(Re^{-5/6})$ as compared to the inertia forces which are $O(\beta)$ or $O(Re^{-1/6})$. Therefore it is assumed that the flow is inviscid and that the perturbation vorticity is convected without sensible diffusion in this region.

Similarly it can be shown from the continuity of mass that the size of the internal wake is,

$$\delta_w = O(\delta^{1/2}).$$

Again from the order of magnitude analysis, it can be shown that the viscous terms in the internal wake are negligible compared to the inertia terms and that the wake is effectively inviscid.

The above arguments imply as mentioned in [10], that the vorticity which is convected passively without diffusion in the region near the rear stagnation point and through the internal wake will turn near the front stagnation point and re-enter the boundary layer. However, it should be noted from the boundary condition given by equation (14c) that there is a discontinuity in W at $Y = 0$ near the front stagnation point ($X = 0$) since $a_0 \neq 0$ in general and this discontinuity will smoothen as the vorticity diffuses in the boundary layer. In Harper and Moore's problem such a discontinuity did not exist because the shear stress at the interface in their problem was symmetrical with respect to $\theta = \pi/2$, which is not so in our case. The above condition is used to determine the unknown function $g(Y)$ and K_1 in equation (16). Mathematically this condition is,

$$W(0^+, Y) = W(X_e^-, Y) = g(Y). \tag{17}$$

This condition when substituted in equation (16) leads to an integral equation similar to equation (3.23) in [10]. Therefore,

$$\frac{1}{2(\pi X_e)^{1/2}} \int_0^Y g(Y')K(Y, Y')dY' - 3\phi_a(X_e, Y) + K_1\phi_b(X_e, Y) = g(Y) \tag{18}$$

where

$$K(Y, Y') = \exp[-(Y - Y')^2/4X_e] - \exp[-(Y + Y')^2/4X_e].$$

This equation is subjected to the restriction that $g(Y \rightarrow \infty) = 0$.

This integral equation was solved by a numerical procedure given in Appendix B and the function $g(Y)$ and constant K_1 which represents the strength of the Hill's vortex was determined.

It was found that the value of K_1 is,

$$K_1 = \frac{U_\infty}{AR^2} = O(4-10).$$

The value of K_1 was found to be between 4 and 10 depending upon the stress imposed and the liquid viscosity chosen. The important point to note is that a similar ratio of the free stream velocity with respect to the droplet to the characteristic velocity in the liquid, that is, U_∞/AR^2 is of the order of one for the Harper and Moore's [10] problem, or a difference of about an order of magnitude from our value. The reason for this is that the density and viscosity of the free stream fluid and the droplet in their problem are of the same order which is not so in our problem.

Once $g(Y)$ and K_1 are known, the solution in the boundary layer can be determined using equations (16) and (11).

Now we would like to examine whether ω_1 is in fact of the order of one in the boundary layer, as was assumed for the analysis. For this purpose we would like to estimate the value of ω_1 at the droplet surface.

The shear stress at the surface of a sphere can be expressed as,

$$\text{shear stress} = O(Re_q^{1/2}\mu_q U_\infty \sin \theta/R).$$

Now the continuity of shear stress at the gas-liquid interface would require,

$$\left. \frac{\partial q'_1}{\partial r'} \right|_{r'=1} = -3 \sin \theta + O[Re_q^{1/2}(\mu_g/\mu_l)(U_\infty/AR^2) \sin \theta].$$

Here primes denote the non-dimensional quantities and AR^2 is used to non-dimensionalize velocity.

The quantity on the LHS of the equation is just ω'_1 at the interface. In the second term on the RHS, Re_q is of the order of 200, μ_g/μ_l is typically of the order of 1/30 and for a conservative value of U_∞/AR^2 as 10, this term is of $O(5 \sin \theta)$. Therefore,

$$\omega'_1 = O(1),$$

and the linearization of the momentum equation and the following analysis is self-consistent.

3. ENERGY EQUATION

Since the Prandtl number in the liquid is typically of $O(10-20)$, the Peclet number based on the liquid droplet motion considered above is, $Pe_l = O(1000)$. With this high Peclet number in the liquid, we would expect a thin thermal boundary layer near the droplet surface. The thickness of the thermal layer is, $\delta_t = O(Pe_l^{-1/2})$.

Once the strength of the Hill's vortex is known, the characteristic residence time along a closed streamline can be estimated. For a droplet of diameter $50 \mu\text{m}$ and free stream relative gas velocity of 25 m/s which gives a characteristic liquid velocity of 2.5 m/s , this time is,

$$\tau_{\text{residence along a closed streamline}} = O(D/U_l) = O(20 \mu\text{s}).$$

Similarly the thermal diffusion time for the fuel (ethanol) droplet of $25 \mu\text{m}$ radius at 298 K can be estimated.

$$\tau_{\text{thermal diffusion}} = R^2/\alpha_l.$$

This is of the order of 10 ms in the absence of any vortex motion. In the presence of convective vortex motion, this characteristic thermal diffusion time will reduce by an order of magnitude to about 1 ms for the same reasons as discussed in Section 2 with regard to the vorticity diffusion time.

The droplet lifetime as calculated in Section 2 is of the order of 2 ms which is comparable to the thermal diffusion time. Therefore, the droplet heating will be essentially unsteady. Also since the residence time along a closed streamline which is also the residence time in the thermal boundary layer is about two orders of magnitude smaller than the thermal diffusion time, the thermal boundary layer is essentially quasi-steady. This short residence time along a closed streamline also implies that in the thermal core away from the boundary layer, the temperature variation normal to the streamline will be important. The temperature

variation along any streamline will uniformize and an average value over a closed streamline is used. This would simplify the energy equation in the core where the unsteady one-dimensional energy equation in streamline co-ordinates can be used as was done by Brignell [12] for his solute extraction problem. Coupling between the thermal boundary layer and the core is achieved by requiring the continuity of the gradient of the average temperature normal to the outer streamline for the core.

(A) Thermal boundary layer

The steady state energy equation for the incompressible fluid can be written as follows:

$$\bar{q} \cdot \nabla T = \alpha_i \nabla^2 T. \quad (19)$$

In the above, α_i is assumed to be independent of temperature.

In the above equation, the various quantities are non-dimensionalized similar to the vorticity equation. In particular, we define

$$T' = \frac{T - T_c(t)}{T_{b.p.} - T_0}$$

where $T_c(t)$ is the average temperature at the outer boundary of the core (or the average temperature at the boundary streamsurface) and is a function of time. Here, the boiling point of the liquid is chosen for non-dimensionalization because it is a constant upper bound for the surface temperature. Further define,

$$\delta_i^2 = 1/Pe_i = \alpha_i/(AR^3)$$

$$R - r = R\delta_i y_i$$

retaining terms of $O(1)$ in the equation (19), the following form of the energy equation is obtained.

$$\frac{\partial^2 T}{\partial y_i^2} + 2y_i \cos \theta \frac{\partial T}{\partial y_i} - \sin \theta \frac{\partial T}{\partial \theta} = 0. \quad (20)$$

Here, primes for the non-dimensional temperature have been deleted for convenience.

Transforming the independent variables θ and y_i to X_i and Y_i as was given for the viscous boundary layer by equations (12b) and (12c), we get the well known form of the diffusion equation given below.

$$\frac{\partial^2 T}{\partial Y_i^2} = \frac{\partial T}{\partial X_i}. \quad (21)$$

The boundary conditions for this equation are similar to that for the viscous boundary layer. They are,

$$\text{at } Y_i = 0, \quad \frac{\partial T}{\partial Y_i} = f(X_i, t) \quad (21a)$$

$$\text{at } Y_i \rightarrow \infty, \quad \frac{\partial T}{\partial Y_i} = C_1. \quad (21b)$$

Here $f(X_i)$ is the temperature gradient along the droplet surface and is obtained by coupling with the gas phase boundary-layer solution. This represents the heat flux entering the droplet from the gas side and can be estimated in similar fashion to the shear stress when the droplet is not sufficiently heated and the vaporization rate is small. Note that the heat flux will be time

dependent for the vaporization problem due to the temporal variation of surface temperature and thus the vaporization rate. The parameter C_1 is related to the average temperature gradient normal to the streamline at the bounding streamline for the core region and will, similarly, be time dependent for the vaporization problem.

If we define,

$$V = \frac{\partial T}{\partial Y_i} - C_1,$$

then equation (21) and the boundary conditions given above and valid at any instant, can be written as,

$$\frac{\partial^2 V}{\partial Y_i^2} = \frac{\partial V}{\partial X_i} \quad (22)$$

$$\text{at } Y_i = 0, \quad V = f(X_i) - C_1 \quad (22a)$$

$$\text{at } Y_i \rightarrow \infty, \quad V = 0. \quad (22b)$$

Another boundary condition at some value of X_i is needed to solve equation (22). Let,

$$\text{at } X_i = 0^+, \quad V = h(Y_i). \quad (22c)$$

Here $h(Y_i)$ is a function similar to $g(Y)$ in Section 2 and has to be determined.

Using arguments similar to that for ω_1 , it can be shown that in the turning region near the rear stagnation point and in the internal wake, there is negligible diffusion of heat and that the temperature gradient V is convected without diffusion through this region to re-enter the thermal boundary layer. This condition leads to an equation similar to (17).

$$V(0, Y_i) = h(Y_i) = V(X_e, Y_i). \quad (23)$$

The condition given by equation (23) along with the differential equation (22) with boundary conditions (22a) and (22b) imply that the integral of $f(X_i)$ over the droplet surface which represents the total heat flux into the droplet will equal the heat flux across the portion of the bounding streamsurface near the droplet surface on which a constant average temperature gradient exists. We thereby assume that the flux of heat from the part of the bounding streamsurface along the internal wake is small compared to the total heat flux which is true because of the small fraction of the surface area for this part of the streamsurface. Thus the value of C_1 could be determined if $f(X_i)$ is known. That is,

$$C_1 = \frac{1}{X_e} \int_0^{X_e} f(X_i) dX_i.$$

The condition given by equation (23) will lead to an integral equation for $h(Y_i)$ similar to the one given by equation (18). This integral equation subject to the condition $h(Y_i \rightarrow \infty) = 0$, can be solved and $h(Y_i)$ and C_1 determined similar to the way $g(Y)$ and K_1 were determined. Once these quantities are known, solution in the thermal boundary layer can be determined.

(B) Thermal core

Mathematical analysis for the core is similar to that in [12] except for the difference in nomenclature.

The unsteady energy equation valid in the core is,

$$\frac{\partial T}{\partial t} + \bar{q} \cdot \nabla T = \alpha_l \nabla^2 T. \tag{24}$$

Because of the short residence time along a closed streamline compared to the thermal diffusion time, this equation is integrated along a closed streamline and an average temperature on that streamline is defined as,

$$T_{av}(\psi, t) = \frac{\oint \frac{T}{q_1} \cdot h_z d\xi}{\oint \frac{1}{q_1} \cdot h_z d\xi}.$$

We further define the non-dimensional quantities as follows.

$$\left. \begin{aligned} \phi &= -8\psi/(AR^4) = 4p^2(1-p^2)\sin^2\theta \\ g_1(\phi) &= \frac{AR}{8} \oint \frac{h_z}{q_1} d\xi \\ g_2(\phi) &= \frac{8}{AR^5} \oint \frac{h_z}{h_v} \cdot h_z d\xi. \end{aligned} \right\} \tag{25}$$

Here subscript "o" has been dropped for the Hill's vortex solution and the various terms used have been defined in the nomenclature. We further non-dimensionalize temperature and time as follows:

$$\begin{aligned} T' &= \frac{T_{av} - T_0}{T_{b.p.} - T_0} \\ \tau' &= \alpha_l t / R^2. \end{aligned}$$

Now equation (24) can be written in streamline coordinates, deleting primes for non-dimensional temperature and time as,

$$\frac{\partial T}{\partial \tau} = \frac{1}{g_1(\phi)} \frac{\partial}{\partial \phi} \left[g_2(\phi) \frac{\partial T}{\partial \phi} \right]. \tag{26}$$

This equation is identical to equation (16) in [12]. We have the initial condition,

$$(1) \quad \text{at } \tau = 0, \quad T = 0, \tag{26a}$$

since the core is initially at T_0 .

The boundary $\phi = \phi_o$ corresponds to the outer boundary of the core near the droplet surface. Its value can be estimated from the thermal boundary layer thickness. It was found that $\phi_o \approx 0.16$. We have the following boundary condition at this end.

$$(2) \quad \text{at } \phi = \phi_o, \quad \frac{\partial T}{\partial \phi} = C_2, \tag{26b}$$

where C_2 is a constant obviously related to C_1 used in equation (21b). The boundary condition at the center of the vortex is obtained by assuming that T is a regular function of ϕ at this point. Thus the condition obtained is,

$$(3) \quad \text{at } \phi = 1, \quad \frac{\partial T}{\partial \tau} = \left[\frac{1}{g_1(\phi)} \frac{dg_2}{d\phi} \right]_{\phi=1} \cdot \frac{\partial T}{\partial \phi}. \tag{26c}$$

This condition is identical to equation (18) in [12]. Since equation (26) and the boundary conditions are linear, the solution to the above problem can be obtained by solving the problem with normalized

boundary condition at $\phi = \phi_o$.

In addition to the gradient boundary condition, the above equation was also solved for $T = 1$ at $\phi = \phi_o$, which is relevant when the surface temperature has reached a value close to the boiling point and the droplet core is still heating. The results of the above solution are similar to the results shown in Fig. 5 of [12]. These solutions were obtained primarily to demonstrate that the thermal diffusion time is significantly reduced by about an order of magnitude because of the vortex motion.

The main difference between this problem and the Brignell's [12] problem is in the coupling between the thermal core and the boundary layer in this problem. Brignell could neglect the existence of the concentration boundary layer without causing any significant error in the mass-transfer problem. Although the temperature change across the thermal layer is of $O(Pe_l^{-1/2})$ (in the non-dimensional sense) and the neglect of the thermal layer would cause only a small error of the same order in determining the temperature field in the core, such a coupling is essential in our ultimate problem because the fuel vapor fraction and hence the vaporization rate, which in turn determines the heat flux, depends exponentially on the surface temperature.

To determine the relation between C_1 and C_2 and thus the coupling between the core and the thermal boundary layer, we have in the thermal layer,

$$\psi = -\frac{1}{2} Ar^2 (R^2 - r^2) \sin^2 \theta.$$

therefore

$$\psi \approx -AR^4 \delta_r y_l \sin^2 \theta.$$

Using the definitions of δ_r , y_l and Y_l and taking into account the area ratio of the bounding streamsurface to its part along the interface, we find,

$$\left[\frac{\partial T}{\partial Y_l} \right]_{Y_l = \delta_r} = C_1 = -AR^4 \delta_l \times \left[\frac{\partial T}{\partial \psi} \right]_{\text{at the core boundary}} (1 + \delta_{w_c}/R).$$

Therefore,

$$C_1 = 8\delta_l C_2 (1 + \delta_{w_c}/R)$$

where

$$\delta_{w_c}/R = O(Pe_l^{-1/4}).$$

It will be more meaningful to solve the coupled quasi-steady thermal boundary layer equation and the unsteady core equation, when the gas phase coupling is done and the vaporization is taken into account because as the core gets heated, the droplet surface temperature will rise resulting in an increased vaporization rate and therefore a decrease in heat flux into the core.

4. RESULTS AND DISCUSSION

The diffusion equation (26) was solved numerically for the normalized boundary condition at the end $\phi = \phi_o$. The results for the case of given temperature

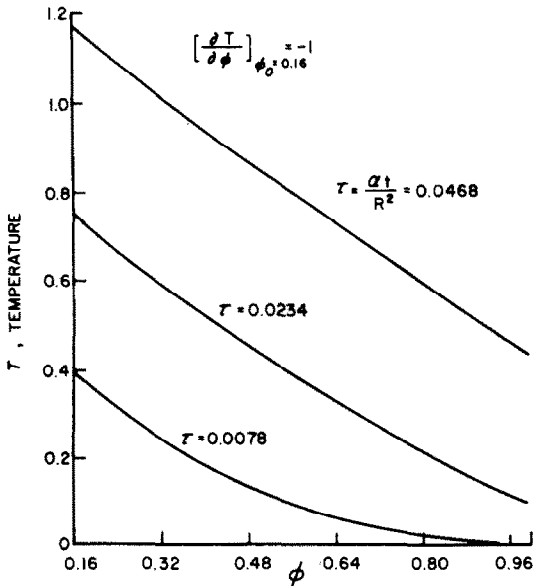


FIG. 3. Average temperature on a closed streamline as a function of ϕ at various instants when $[\partial T/\partial\phi]_{\phi=\phi_0} = 1$.

gradient at the boundary and for the case of given temperature at the boundary are shown in Figs. 3 and 4 respectively. For the first case, a steady-state solution does not exist and the temperature keeps rising, approaching a linear profile. In reality, of course, the surface temperature cannot exceed the boiling point of the liquid so that temperature gradient at the surface would eventually decrease with time. For the latter case, it can be seen from Fig. 4 that for $\tau = 0.09$, the interior is heated to about 92% of the boundary value.

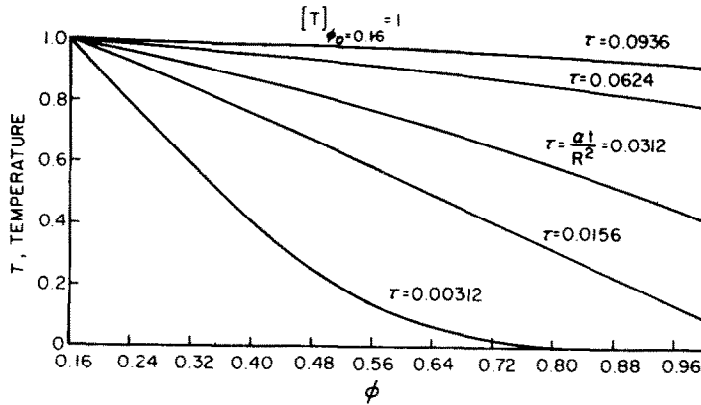


FIG. 4. Average temperature on a closed streamline as a function of ϕ at various instants when $[T]_{\phi=\phi_0} = 1$.

This implies that it takes about 10% of the characteristic thermal diffusion time (R^2/α_1), for the droplet to be fully heated in the presence of convective motion. This was mentioned earlier on physical grounds. Similarly, it can be observed from Fig. 3 that in about 5% of the characteristic time (R^2/α_1), the temperature profile becomes almost linear.

In the present paper, the motion generated inside a droplet in a gaseous environment has been studied

using Harper and Moore's approach which was primarily developed for a droplet motion in a fluid of comparable density and viscosity. It is found that the droplet motion is still a Hill's vortex whose strength is different by about an order of magnitude from the Harper and Moore vortex and that a thin boundary layer near the droplet surface exists. The same approach for the droplet motion can be used in the presence of vaporization when the gas phase boundary layer is coupled. Although in that case continuity of tangential velocity will be required in addition to the continuity of shear stress at the interface which will necessitate an iterative solution of the gas-liquid boundary layers. The quasi-steady assumption for the liquid motion will still hold although the change in droplet radius with time will have to be taken into account.

Because of a short residence time along a closed streamline in the core for vortex strength in the range of interest, the above analysis shows that the heating time of the core will be independent of the strength of the vortex and hence the motion generated inside the droplet. Thermal diffusion will be essentially normal to the streamlines. The vaporization rate and the heating of the core are coupled through the thermal boundary layer which determines the surface temperature and therefore the vaporization rate and consequently the heat flux into the core. In the gas-liquid coupled problem with vaporization, the energy equation in the thermal core has to be modified to take into account the temporal variation of ψ and the radius of the droplet. But the averaging process along a stream surface will still hold for the same reasons as here.

In the droplet vaporization problem with convective field, it is seen that the unsteady effects will persist due to droplet heating for the major part of the droplet lifetime even though the internal circulation will significantly enhance the heating as shown in this paper. The rapid mixing or uniform temperature assumption therefore does not lead to an accurate prediction of either the temporal or spatial dependence of temperature.

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APPENDIX A

The function $\phi_b(X, Y)$ is a solution of the diffusion equation corresponding to the second part of the boundary condition at $Y = 0$, that is, $(a_0 + a_1X + a_2X^2 + a_3X^3)$ and is given as follows:

$$\begin{aligned} \phi_b(X, Y) = & a_0 \operatorname{erfc}[Y/(2X^{1/2})] + 4a_1XY \operatorname{erfc}[Y/(2X^{1/2})] \\ & + 32a_2X^2i^2 \operatorname{erfc}[Y/(2X^{1/2})] \\ & + 384a_3X^3i^6 \operatorname{erfc}[Y/(2X^{1/2})] \\ & + H(X - X_0)b_0 \operatorname{erfc}[Y/2(X - X_0)^{1/2}] \\ & + 4b_1(X - X_0)i^2 \operatorname{erfc}[Y/2(X - X_0)^{1/2}] \\ & + 32b_2(X - X_0)^2i^4 \operatorname{erfc}[Y/2(X - X_0)^{1/2}] \\ & + 384b_3(X - X_0)^3i^6 \operatorname{erfc}[Y/2(X - X_0)^{1/2}], \quad (A1) \end{aligned}$$

Here $H(X - X_0)$ is the Heaviside step function which is zero for $X < X_0$ and equal to one for $X > X_0$. The functions i, i^2, i^4, \dots etc. are integrals of complementary error functions defined elsewhere [17]. The constants b_0, b_1, \dots etc. are related to a_0, a_1, \dots etc. as follows.

$$\begin{aligned} b_0 = & -(a_0 + a_1X_0 + a_2X_0^2 + a_3X_0^3) \\ b_1 = & -(a_1 + 2a_2X_0 + 3a_3X_0^2) \\ b_2 = & -(a_2 + 3a_3X_0) \\ b_3 = & -a_3. \end{aligned} \quad (A2)$$

The function $\phi_b(X, Y)$ could also be calculated numerically. This may be necessary if a good polynomial fit without many terms is not possible.

APPENDIX B

Equation (18) was solved by a procedure similar to that in [10]. We write

$$g(Y) = 3g_a(Y) - K_1g_b(Y), \quad (B1)$$

and require that $g_a(Y)$ must satisfy

$$\frac{1}{2(\pi X_c)^{1/2}} \int_0^Y g_a(Y')K(Y, Y')dY' - g_a(Y) = \phi_a(Y) \quad (B2)$$

and a similar equation for $g_b(Y)$ with b replaced by c . The above two integral equations are equivalent to the original equation (18). The solutions of the above two equations are rendered unique by requiring that $g_b(Y)$ and $g_c(Y)$ have a finite limit as $Y \rightarrow \infty$.

Equation (B2) was solved iteratively. An initial guess, $g_0(Y)$, for $g_a(Y)$ was made at equally spaced intervals of Y and the integral was evaluated numerically using Simpson's rule. Thus a new set of values $g_1(Y)$ is obtained from equation (B2). A new guess, $g_2(Y)$, for $g_a(Y)$ is obtained by combining the old set of values and the new set linearly. Thus,

$$g_2(Y) = (1+k)g_1(Y) - kg_0(Y).$$

This process is repeated by replacing $g_0(Y)$ by $g_2(Y)$ in the integral until convergence is obtained. Value of k , a constant, is chosen to get the fastest convergence. The best value was found to be about 0.6.

Once $g_a(Y)$ and $g_b(Y)$ are known, K_1 is determined from the condition $g(Y \rightarrow \infty) = 0$. Therefore,

$$K_1 = 3g_a(\infty)/g_b(\infty).$$

CHAUFFAGE D'UNE GOUTTE LIQUIDE COMBUSTIBLE AVEC UNE CIRCULATION INTERNE

Résumé—On étudie la circulation interne d'une goutte liquide en déplacement dans un gaz pour des paramètres caractérisant un mélange dispersé air-combustible dans un cas typique. L'analyse montre un mouvement axisymétrique et quasi-statique du liquide avec un tourbillon sphérique central entouré par une couche limite visqueuse et un sillage interne. L'intensité du tourbillon est déterminé en liaison avec la tension de cisaillement sur la surface de la goutte. De plus l'analyse indique que le chauffage de la goutte est variable durant sa durée de vie. La circulation interne rend effectivement monodimensionnelle la conduction thermique dans le noyau liquide. La couche limite thermique près de la surface est étudiée et on montre que le champ de température dans la goutte peut être déterminé à partir de la contrainte à l'interface gaz-liquide.

HEIZUNG MITTELS FLÜSSIGER ÖLTRÖPFCHEN MIT INNERER STRÖMUNG

Zusammenfassung—Die innere Strömung eines flüssigen Tröpfchens, das sich durch ein Gas bewegt, wird in einem Parameterbereich analysiert, der für ein brennendes Luft-Brennstoff-Gemisch in einer typischen Brennkammer maßgebend ist. Die Analyse zeigt eine axisymmetrische quasistationäre Flüssigkeitsbewegung mit einem sphärischen Kernwirbel, der durch eine zähe Grenzschicht und eine Wirbelschlepe umgeben ist. Die Wirbelstärke wird als eine Funktion der Schubspannung über der Oberfläche des Tröpfchens berechnet. Außerdem zeigt die Analyse, daß die Erwärmung des Tröpfchens während der Zeit seines Auftretens instabil ist. Es zeigt sich, daß die innere Strömung den Wärmetransport durch Leitung im Flüssigkeitskern wirkungsvoll in eine Richtung fließen läßt. Die thermische Grenzschicht in der Nähe der Oberfläche wird ebenfalls analysiert, und es wird gezeigt, daß das Temperaturfeld des Tröpfchens durch die Angabe der Gas-Flüssigkeit-Grenzbedingungen berechnet werden kann.

НАГРЕВ КАПЛИ ЖИДКОГО ТОПЛИВА ПРИ НАЛИЧИИ ВНУТРЕННЕЙ ЦИРКУЛЯЦИИ

Аннотация — Анализ внутренней циркуляции капли жидкости, проходящей через газ, проводится в диапазоне параметров, характерных для горения смеси распыленного топлива с воздухом в типичной камере сгорания. В результате анализа выявлено осесимметрично квазистационарное течение жидкости, в котором имеет место сферическое вихревое ядро, окруженное вязким пограничным слоем и внутренним следом. Интенсивность вихря определяется как функция касательного напряжения на поверхности капли. Кроме того выяснено, что нагрев капли является нестационарным за время жизни капли. Показано, что внутренняя циркуляция ведет к одномерности теплопроводности в ядре. Также анализируется тепловой пограничный слой у поверхности капли и показано, что температурное поле капли может быть определено, если известны условия на границе раздела газ-жидкость.